AP Calculus AB
Test: Applications of the Integral
Instructor's Version

Full Name:_____ **User Name:**_____ **Date:**_____
(print clearly)

Free-Response Question 1 (no calculator)
You may not use your calculator for this question.
(When you finish this question, if your test is proctored; ask your proctor for the next section. If you're taking the unproctored version of the test, continue with the next question.)

1. The region R is bounded by the x-axis, $x = 1$, $x = 3$, and $y = \frac{1}{x^3}$.

A. Find the area of R.

Answer: $\int_1^3 \frac{1}{x^3}\,dx = \left[\frac{-1}{2x^2}\right]_1^3 = \frac{4}{9} = 0.444$

1 point for integrand;
1 point limits of integration
1.75 points for answer

Note: A decimal answer within ± 0.001 is acceptable.

B. Find the value of h, such that the vertical line $x = h$ divides the region R into two Regions of equal area.

Answer: $\int_1^h \frac{1}{x^3}\,dx = \left[\frac{-1}{2x^2}\right]_1^h = \frac{-1}{2h^2} + \frac{1}{2} = \frac{2}{9}$

$\frac{-1}{h^2} + 1 = \frac{4}{9} \Rightarrow h = \sqrt{\frac{9}{5}} \approx 1.342$

1 point for setting area equal to $\frac{1}{2}$ previous area.

1 point for antiderivative
1.75 points for answer

Note: A decimal answer within ± 0.001 is acceptable. Points may be awarded if student works part B correctly using an incorrect value from part A.

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C. Find the volume of the solid generated when R is revolved about the x-axis.

Answer: $V=\int_1^3 \pi\left(\frac{1}{x^3}\right)^2 dx=\pi\left[\frac{-1}{5x^5}\right]_1^3=\pi\left[\frac{-1}{1215}+\frac{1}{5}\right]\approx 0.626$

1 point for integrand
1 point for limits of integration and π
1.75 points for answer

Note: A decimal answer within ± 0.001 is acceptable.

D. The vertical line $x = k$ divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

Answer: $V(k)=\int_1^k \pi\left(\frac{1}{x^3}\right)^2 dx=\pi\left[\frac{-1}{5x^5}\right]_1^k=\pi\left[\frac{-1}{5k^5}+\frac{1}{5}\right]=0.313$

$k\approx 1.148$

1 point for setup of integral
1 point for antiderivative
1.75 point for answer

Note: If students use an incorrect volume from part C to solve for k, give points if done correctly using that value. Calculator answers within ± 0.001 are acceptable.

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Free-Response Question 2 (no calculator)
You may not use your calculator for this question.

2. River currents (miles per hour) at a certain location are given below. The current direction at this location was "from the north" during the time interval shown, and the current did not exhibit any severe fluctuations other than those shown in the chart.

Time of Day	6:00 AM	6:10 AM	6:20 AM	6:30 AM	6:40 AM
Speed (mph)	17	20	22	21	17

A. Using trapezoids, estimate the average wind speed from the north from 6:00 AM until 6:40 AM.

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{b-a}\bullet\frac{\Delta x}{2}\left(f(x_1)+2f(x_2)+2f(x_3)+2f(x_4)+f(x_5)\right) =$$

Answer: Avg = **but** $b-a=40 \text{ min} = \frac{2}{3}$ **hour;** $\Delta x = \mathbf{10\ min} = \frac{\mathbf{1}}{\mathbf{6}}$ **hour**

$$= \frac{3}{2}\bullet\frac{1}{12}(17+2(20)+2(22)+2(21)+17) = 20\ mph$$

2 points setup
2 points answer
1 point units

B. At approximately what time between 6:00 AM and 6:40 AM would you estimate that the wind had the average velocity?

Answer: At 6:10 AM (2 points)
Note: Students may get this point even with a wrong answer from part A if they estimate the time correctly based on their average velocity from part A.

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C. A message in a bottle near this location is released at 6:00 AM. Assuming that the bottle travels along with the river's current, approximately how far south will the bottle be at 6:40 AM?

$$\textbf{Distance} = \int_{6:00}^{6:40} v(t)\,dt \approx \frac{\Delta x}{2}\left(f(x_1)+2(f(x_2))+2(f(x_3))+2(f(x_4)+f(x_5))\right)$$

Answer: $= \frac{1/6}{2}(17+2(20)+2(22)+2(21)+17) = \frac{40}{3}$ **miles**

OR $\textbf{Distance} = \textbf{Average Velocity} \bullet \textbf{time} = \textbf{20 mph} \bullet \frac{\textbf{2}}{\textbf{3}}\ \textbf{hr} = \frac{\textbf{40}}{\textbf{3}}\ \textbf{miles}$

2 points for setup
2 points for answer
1 point for units

Note: If students are correctly using a wrong answer from part A, they should get full credit.

D. What is the average acceleration of this bottle from 6:00 AM to 6:40 AM?

Answer: $\textbf{acceleration} = \frac{\Delta v}{\Delta t} = \frac{\textbf{final velocity} - \textbf{initial velocity}}{\textbf{change in time}} = \frac{17-17}{\frac{2}{3}} = 0\ \frac{\textbf{miles}}{\textbf{hour}^2}$

2 points for answer
1 point for units

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